RAN-2103000206020031

T. Y. B. Sc. (Sem. -VI) Examination April - 2023

Mathematics : Paper - MTH - 601

Ring Theory

Time: 2 Hours] **[Total Marks: 50**

સૂચના : / Instructions

(1) નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fill up strictly the details of ☛ signs on your answer book

Name of the Examination:
☛ **T. Y. B. Sc. (Sem. -VI)**

Name of the Subject :
☛ **Mathematics : Paper - MTH - 601 Ring Theory**

Subject Code No.: **2103000206020031**

Seat No.:

Student's Signature

(2) Figures to the right indicate marks of corresponding question.
(3) Follow usual notations.
(4) Use of non-programmable scientific calculator is allowed.

Q. 1. Answer the following as directed : (Any FIVE) (10)

(1) Define Ring with a *Unit* element. Give an example of Ring without a *Unit* element.

(2) In a Boolean ring R; prove that $a + a = 0$; for every a in R.

(3) Let $\phi : R \to R'$ be a homomorphism of a ring R into a ring R'. Then prove that $\phi(-a) = -\phi(a)$; for every a in R.

(4) If U is an ideal of a ring R with a *unit* element 1 and $1 \in U$, then prove that $U = R$.

(5) Justify: 3 is a greatest common divisor of 3 and 6 in the commutative ring $3\mathbb{Z}$.

(6) Let R be a Euclidean ring and $a \neq 0$, $b \neq 0$ in R. If b is *unit* in R, then prove that $d(a) = d(ab)$.

(7) Define Prime Element and Relatively Prime Elements in a Euclidean Ring.

(8) If π is a prime element in a Euclidean Ring R and $\pi \mid ab$; for $a, b \in R$, then prove that $\pi \mid a$ or $\pi \mid b$.

Q. 2. **Answer any Two of the following :** **(10)**

(a) Define Zero Divisor in Commutative Ring. Prove that every field is an integral domain.

(b) Prove that the commutative ring D is an integral domain if and only if $a, b, c \in D$ with $a \neq 0$; the relation $a \cdot b = a \cdot c \Rightarrow b = c$ holds in D.

(c) Define a Boolean ring. Prove that every Boolean ring is commutative.

Q. 3. **Answer any Two of the following :** **(10)**

(a) Let $\phi : R \rightarrow R'$ be a homomorphism of a ring R into a ring R'. Then prove that ϕ is an isomorphism if and only if $I(\phi) = (0)$; where $I(\phi)$ is the Kernel of a homomorphism ϕ.

(b) Define Right Ideal in a Ring. If F is a field, then prove that its only ideals are (0) and F itself.

(c) Let $\phi : R \rightarrow R'$ be a homomorphism of a ring R onto a ring R'. If R is a commutative ring with a *unit* element 1, then prove that R' is also a commutative ring with a *unit* element $\phi(1)$.

Q. 4. **Answer any Two of the following :** **(10)**

(a) Give an example of a Euclidean ring which is not a field. Prove that every field is a Euclidean ring.

(b) (i) Find all *associates* of $\overline{3}$ in the commutative ring J_9; of integers modulo 9; with a *unit* element $\overline{1}$.

(ii) If ; for a, b in an integral domain R with a *unit* element; both $a|b$ and $b|a$ hold true, then prove that $a = u \cdot b$; where u is some *unit* in R.

(c) Define *Associates* in a Commutative Ring with a *Unit* element . Prove that two greatest common divisors of a, b in a Euclidean ring R are *associates*.

Q. 5. **Answer any Two of the following :** **(10)**

(a) Prove that every non-zero element in a Euclidean ring R is either *unit* or can be written as a product of finite number of prime elements in R.

(b) Mention all the prime elements in the Euclidean rings $\mathbb{Z}$ of all integers and J_{17} ; of integers modulo 17. If a, b and c are elements in a Euclidean ring R such that $(a, b) = 1$ and $a|bc$, then prove that $a|c$.

(c) Prove that an element a in a Euclidean ring R is *unit* in R if and only if $d(a) = d(1)$.
